CHAPTER 12

STABILITY OF SLOPES

إذا كان سطح الارض مائلا بفعل العوامل الطبيعية أو بفعل الإنسان سواء بالحفر فى التربة الطبيعية أو بالردم أعلى التربة الطبيعية – فإن كتلة التربة أسفل الميل تكون معرضة لقوى الجاذبية الأرضية إضافة الى القوى الناتجة عن انسياب المياه والهزات الأرضية وخلافه. فإذا تعدت اجهادات القص أسفل كتلة التربة أو خلالها مقاومة القص للتربة تصبح الكتلة غير متزنة فتتحرك منهارة الى أسفل . ويؤدى انهيار الميول الى كارثة للافراد والممتلكات اذا كانت هناك منشات أعلى او أسفل الميل.

وتعتبر دراسة إتزان ميول الأتربة من الدراسات الهامة فى مجال ميكانيكا التربة وتطبيقاتها عديدة فى مجال الحفر والإنشاء بالأماكن غير المستوية.

Earth embankments and cuttings are commonly encountered in civil engineering works, e.g. railways, roadways and earth dams. The stability of embankments and cutting, or generally called slopes, is to be carefully analyzed since failure may cause damage.

The soil mass having a sloping surface is subjected to internal shearing stresses the gravitational force tends to cause the soil mass to "flow" to a nearly level surface. Failure of soil mass in a slope is called a "slide" (إنزلاق).

Natural slopes may fail due to:

(a) Change of stress by:

- (i) adding loads at the top of the slope,
- (ii) increasing of the angle of the slope, or
- (iii) excavating at the toe of the slope

(b) Increase of pore water pressure by rainfall or rise of groundwater table
(c) Decreases shear strength of soil due to weathering factors, migration

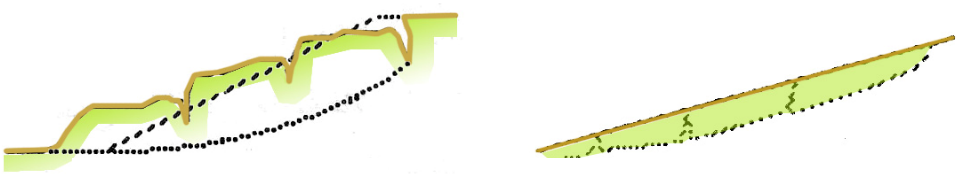
(هجرة) of fine particles by seepage water, change of the mineralogical composition of soil, opening and softening of fissure (شقوق) or progressive increases of shear deformations,

(d) Dynamic effect of earth quakes, explosive or pile driving.

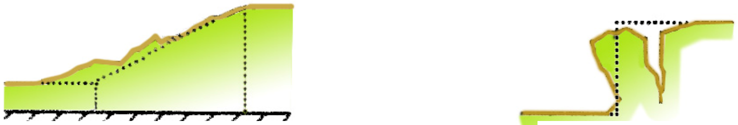
Slopes of embankments may fail due to:

(a) Increase of stress on foundation soil
(b) Sudden drawdown of upstream water level in earth dams
(c) Dynamic effects.

Typical modes (أنماط) of slope failure are shown in Fig. 12.1.



(a) Rotational slip (b) Translational slip in infinite slope



(c) Translational slip on weak interface (d) Rock overturning

(e) Rock fall (f) Flow of soft soil

12.1 Shear strength parameters used for analysis of slopes

12.1 Shear strength parameters used for analysis of slopes

Analysis of slope includes:

(a) Determination of the position of the surface along which shearing stresses are most critical.

(b) Determination of the value of these shearing stresses and comparing them with the shearing resistance of the soil.

The shear strength parameters used in the analysis depend on type of embankment and foundation soil, groundwater condition, rate of loading and weathering effects. These parameters are normally determined based on total stresses, in which pore water pressure in neglected or effective stresses, in which pore water pressure is fully dissipated. Selection of the type of stresses in the analysis depends on the condition of drainage of pore water which in turn depends the permeability of soil and available time for drainage.

12.1.1 Total stress analysis

If pore water has no escape during shear, the undrained shear strength parameter (c_u) determined from vane test, unconfined compressive strength test or undrained trixial test, can be used. This parameter can also be determined from in-situ tests such as in-situ vane test or static cone test. In this analysis the angle of shearing resistance (ϕ) is zero.

Total stress analysis is used in the following case:

(a) Analysis of cuts in normally consolidated clay
(b) Analysis of embankments built on soft clay
(c) Analysis of earth dams in case of sudden drawdown of upstream water level, assuming that the drawdown is quick such that no dissipation(تلاشی) of pore water could occur
(d) Analysis of end of construction of embankments built using cohesive soil.

12.1.2 Effective stress analysis

If pore water can escape during shear, the effective shear strength parameters (c', ϕ') determined from drained triaxial test, or consolidated undrained triaxial test with pore water pressure measurement, can be used.

Effective stress analysis is used in the following cases:

(a) Long term analysis of clay cuts and embankments
(b) Short term analysis of cohesionless cuts and embankments
(c) Sudden drawdown in cohesionless embankments.

12.1.3 Factor of safety in slope stability analysis

The factor of safety in slope stability analysis is defined as the ratio between average resisting shear force along the critical slip pane to the sum of components of those forces acting on the slope, along the critical slip plane. This factor of safety should take into consideration both uncertainty of the true representative soil parameters, and the consequences of failure. For example, stability of earth dams in the vicinity of residential or industrial areas should be analysis using high factor of safety. Table 12.1 gives the recommended values of the factor of safety.

Table 12.1 Recommended values of the factor of safety.

Case	Factor of safety
Permanent loading	1.5
Foundations for buildings	2
Temporary loading	1.25 – 1.3
Accidental loading (seismic loading)	1.1

12.2 Stability analysis of infinite slopes

In this case the potential failure surface is parallel to the surface of the sloping ground and occurs at a relatively small depth compared with the extent of the slope. The slope can then be considered infinite, and the effect of its top and bottom ends can be neglected. Stability analysis of this slope is carried out as follows:

Let (AB) be the surface of a slope, Fig. 12.2. Slope failure occurs along plane (CD) parallel to (AB), at some depth (z), depending on the properties of soil.

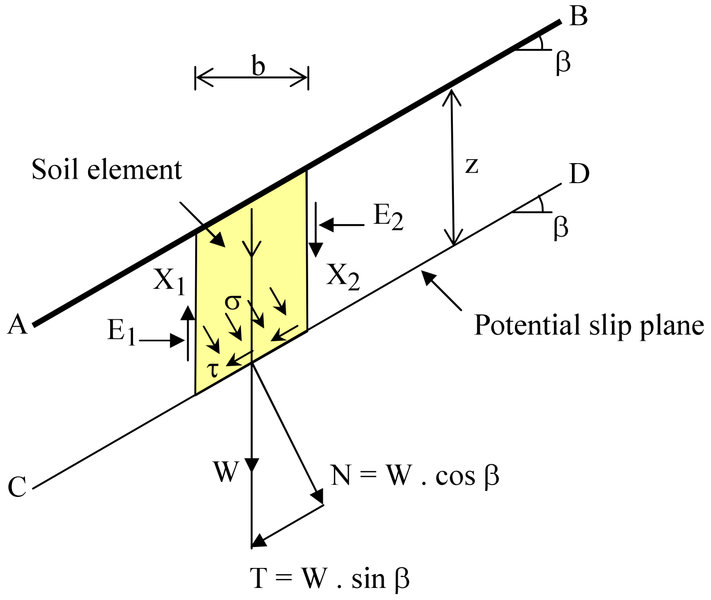


Fig. 12.2 Forces acting on potential slip plane for infinite slope

The forces acting on a soil element of width (b) are:

(a) Self weight (W), which may include any external load,
(b) Normal and shear forces (E1, E2, X1, X2) on the sides,
(c) Normal and shear forces (N,T) on the slip plane.

The forces and stresses on the slip plane are calculated as follows:

$$W = \gamma \cdot z \cdot b$$

$$N = W \cdot \cos\beta$$

$$\sigma = \frac{W \cdot \cos\beta}{\dfrac{b}{\cos\beta}}$$

$$T = W \cdot \sin\beta$$

$$\tau = \frac{W \cdot \sin\beta}{\dfrac{b}{\cos\beta}}$$

$$= \gamma \cdot z \cdot \sin\beta \cdot \cos\beta$$

where:

σ = normal stress on slip plane
τ = shear stress on slip plane

For stability, the shear stress is to be resisted by the shearing resistance of the soil (τ_f). The factor of safety (F.S.) against sliding is given by:

$$F.S. = \frac{\tau_f}{\tau} \qquad \text{… … … … … … … (12.1)}$$

The shearing resistance of soil generally consists of cohesion and internal friction. Both case of cohesionless and cohesive soil are considered.

12.2.1 Infinite slopes in cohesionless soil

In Fig. 12.3, the failure envelope for cohesionless soil is given by:

$$\tau_f = \sigma \cdot \tan\phi$$

where:

σ = normal stress on failure plane
ϕ = angle of internal friction

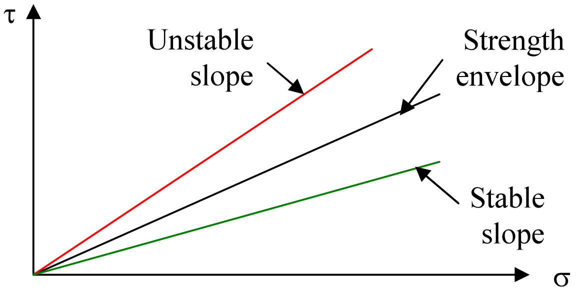


Fig. 12.3 Stability condition for infinite slope in cohesionless soil

Thus:

$$F.S. = \frac{\tau_f}{\tau}$$

$$= \frac{\gamma \cdot z \cdot \cos^2 \beta \cdot \tan \phi}{\gamma \cdot z \cdot \cos \beta \cdot \sin \phi}$$

$$\therefore F.S. = \frac{\tan \phi}{\tan \beta} \qquad \ldots\ldots\ldots\ldots\ldots (12.2)$$

In case the slope is submerged, Fig 12.4, the analysis given above is valid. The submerged weight (W') is used instead of total weight (W). The factor of safety is given by:

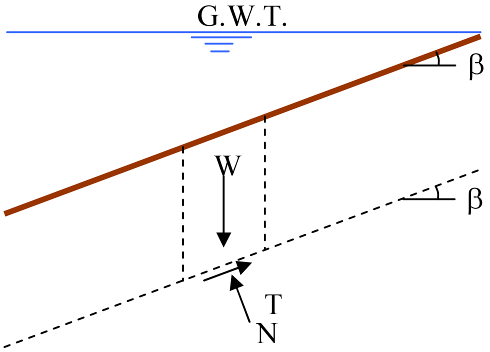


Fig. 12.4 Infinite submergerged slope in cohesionless soil

$$F.S. = \frac{\gamma_{sub} \cdot z \cdot \cos^2 \beta \cdot \tan \phi}{\gamma_{sub} \cdot z \cdot \cos \beta \cdot \sin \phi}$$

$$\therefore F.S. = \frac{\tan \phi}{\tan \beta}$$

When an embankment is formed by end dumping of granular soil from a truck, dropped from a chute or from the end of a conveyor belt, Fig. 12.5, the soil particles will tumble down (تتدحرج) the slope. When the angle of the slope becomes steep, a mass with small thickness will slide down. As stated before in Chapter 11, the angle of inclination is equal to the angle of friction of the soil. The final angle at which the soil stable is called "**angle of repose**".

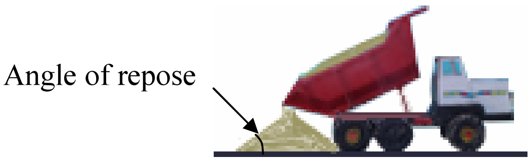


Fig. 12.5 End dumping of granular soil

12.1.2 Seepage parallel to slope

Fig 12.6 shows the case where groundwater is steadily seeping parallel to sloping ground. Flow lines are imagined parallel to the slope, and equi-potential lines will be perpendicular to it.

Considering the shown soil element:

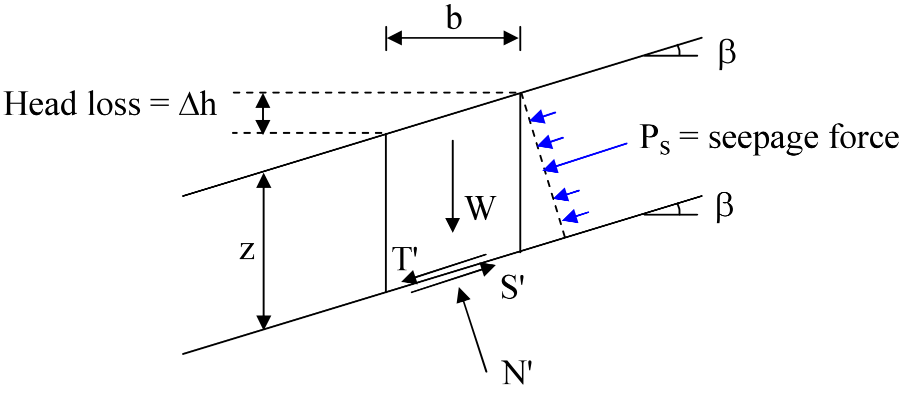


Fig. 12.6 Seepage parallel to slope

$$W' = b.z.\gamma_{sub.}$$

$$N' = b.z.\gamma_{sub.}.\cos\beta$$

$$\Delta h = b.\tan\beta$$

$$Ps = b.\tan\beta.\gamma_{W}.z.\cos\beta = b.z.\gamma_{W}.\sin\beta$$

$$T' = b.z.\gamma_{sub.}.\sin\beta + b.z.\gamma_{W}.\sin\beta = b.z.\gamma_{sat.}.\sin\beta$$

$$S' = b.z.\gamma_{sub.}.\cos\beta.\tan\phi'$$

$$F.S. = \frac{b.z.\gamma_{sub.}.\cos\beta.\tan\phi'}{b.z.\gamma_{sat.}.\sin\beta}$$

$$\therefore\ F.S. = \frac{\tan\phi'}{\tan\beta}\cdot\frac{\gamma_{sub.}}{\gamma_{sat.}} \qquad \text{.. .. .. (12.3)}$$

Since the submerged density of sand is approximately half the saturated density, Equ. 12.3 indicates that the factor of safety in case of seepage parallel to the slope is approximately half that without seepage.

12.1.3 Infinite slopes in cohesive soil

In Fig. 12.7 the failure envelope is given by:

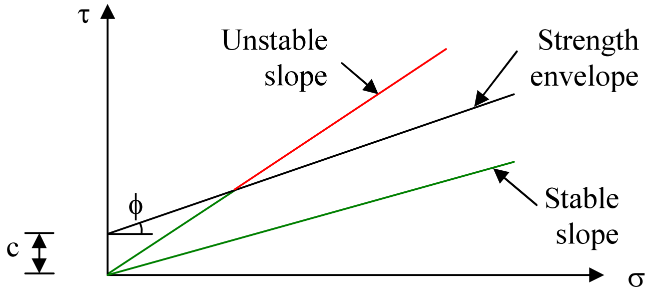


Fig. 12.٧ Stability condition for infinite slope in cohesive soil

$$\tau_f = c + \sigma\tan\phi$$

where : c = cohesion

$$F.S. = \frac{c + \sigma\tan\phi}{\tau}$$

$$= \frac{c + \gamma\, z\cos^2\beta\tan\phi}{\gamma\, z\cos\beta\sin\phi} \qquad \text{.. .. .. (12.4)}$$

12.3 Stability analysis of finite slope

Failure of finite slopes occur along a curved surface. Two types of failures may occur, Fig. 12.8:

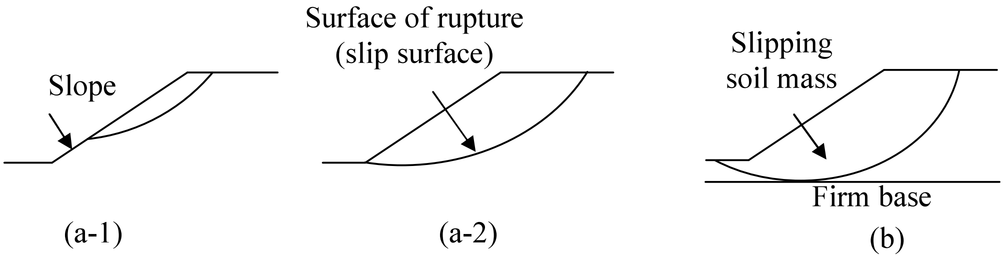


Fig. 12.8 Types of failure of infinite slopes

(a) Slope failure, which is divide into:

(i) Face failure
(ii) Toe failure

(b) Base failure

Methods of analysis of finite slopes are given in the following sections.

12.4 The circular arc method

The surface of sliding in section is assumed to be a circular arc. For fully saturated clay under undrained conditions (ϕ_u = o), i.e. for the conditions immediately after construction, the following analysis is carried out:

An arbitary arc (AD) is chosen, Fig. 12.9. The weight of sliding wedge (W) and distance (x) from is C.G. to the center of rotation (O) are determined. The driving moment (MD) is given by:

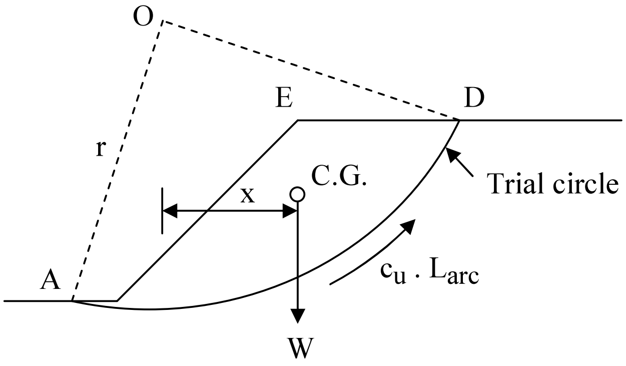


Fig. 12.9 Stability analysis for $\phi = 0$ soil

$$M_D = W \,.\, x$$

The resisting moment (M_R) results from the shear resistance of the soil developed along the slip surface:

$$M_R = c_u \,.\, L_{arc} \,.\, r$$

Therefore, the factor of safety (F.S.) is given by:

$$F.S. = \frac{\text{Resisting moment}}{\text{Driving moment}} = \frac{c_u \cdot L_{arc} \cdot r}{W \cdot x} \qquad \text{… … … … … (12.5)}$$

Assuming that (c_m) is the mobilized shear strength along the failure surface, then:

$$c_m . L_{arc} . r = W.x = \frac{c_u . L_{arc} . r}{F.S.}$$

$$\therefore \; F.S. = \frac{c_u}{c_m} \qquad \text{… … … … … (12.6)}$$

If soil consists of successive layers of different values of shear strength, Fig. 12.10, the analysis can be similarly carried out. The factor of safety will be given by:

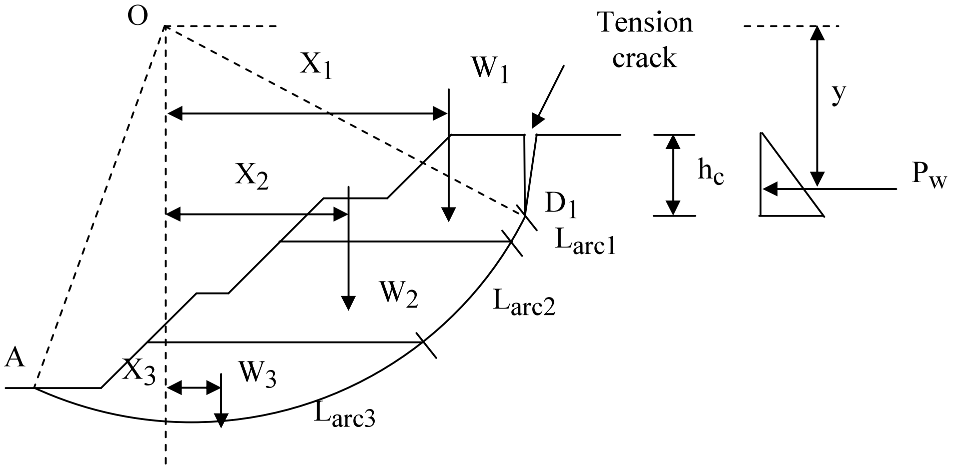


Fig. 12.10 Analysis of layered ϕ = 0 soil

$$F.S. = \frac{r \,.\, \sum\{c.L_{arc}\}}{\sum\{W.x\}} \qquad \text{… … … … … (12.7)}$$

In cohesive soil, a tension crack may appear at some distance from the top of earth slope and parallel to it. In such case, the slip circle will pass through the lower end of the crack. The total length of the arc along which soil shearing resistance develops will be limited to AD_1. Depth of tension crack in purely cohesive soil ($\phi = 0$) is given by:

$$h_c = \frac{2\,c_u}{\gamma} \qquad \text{… … … … … (12.8)}$$

If the tension crack is filled with water, it will exert hydrostatic pressure on the sliding wedge and its moment about (O) will be considered. The factor of safety in such a case is given by:

$$F.S. = \frac{r \cdot \sum\{c\,.\,L_{arc}\}}{P_W\,.\,y + \sum\{W\,.\,x\}} \qquad \text{… … … … … (12.9)}$$

In all case, a series of slip circles are checked, and the one with the lowest factor of safety is the critical one.

12.5 The method of slices

In this method the potential slip surface is assumed to be a circular arc. The soil mass above the assumed slip surface is divided into a number of vertical slices. The factor of safety is defined as the ratio between the shear resistance of soil and mobilized shear stress to maintain equilibrium. The factor of safety is considered to be equal for all slices.

In Fig. 12.11, the weight (W) of each slice of width (b) is resolved into normal force (N) and tangential force (T). The forces on the sides are (E1), E2), (X1) and (X2), however, these forces are usually neglected. But, if any other external forces act on the slope, they should be considered in the analysis.

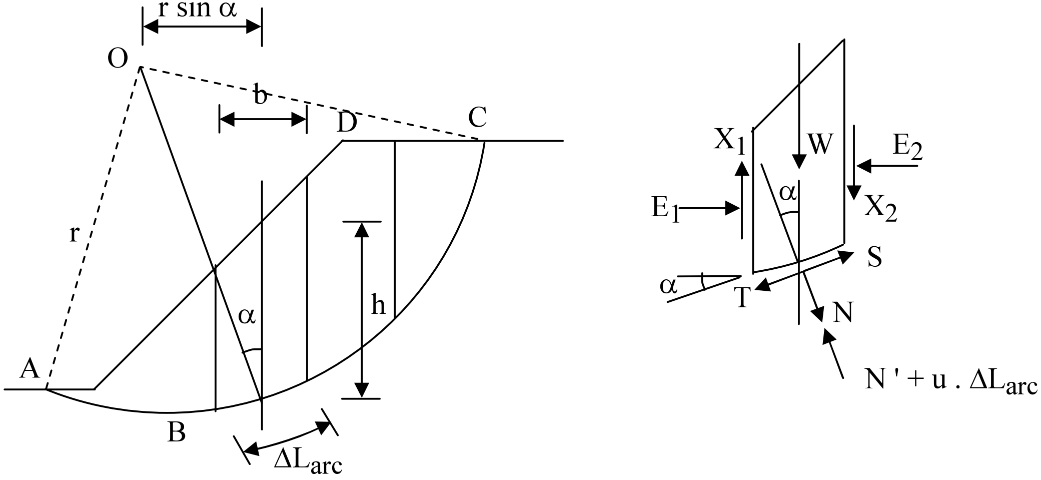


Fig. 12.11 Analysis of layered c – ϕ soil

The driving moment about (O) is:

$$M_D = \sum T \cdot r$$

The resisting shear force along a slice slip surface is given by:

$$S = c \cdot \Delta L_{arc} + N \cdot \tan \phi$$

The resisting moment about (O) is:

$$\begin{aligned} M_R &= \sum (c \cdot \Delta L_{arc} + N \tan \phi) \cdot r \\ &= \left\{c \cdot L_{arc} + \sum N \cdot \tan \phi\right\} \cdot r \end{aligned}$$

$$\therefore \text{F.S.} = \frac{c \cdot L_{arc} + \sum N \cdot \tan \phi}{\sum T} \qquad \ldots\ldots\ldots\ldots\ldots (12.10)$$

12.5.1 The Fellenius solution

In this solution the resultant forces between the slices are neglected. The normal component consists of two parts, effective normal forces (N') and water forces (U) which equal the pore water pressure times length of base arc, Fig. 12.11. For each slice, resolving forces normal to the base:

$$N' = W \cdot \cos \alpha - u \cdot \Delta L_{arc}$$

The factor of safety in terms of effective stresses is given by:

$$\text{F.S.} = \frac{c' \cdot L_{arc} + \left\{\sum (W.\cos \alpha - u \Delta L_{arc})\right\}}{\sum \left\{W.\sin \alpha\right\}} \qquad \ldots\ldots\ldots\ldots\ldots (12.11)$$

In terms of total stress, (c_u) and (ϕ_u) will be used, and (u) is zero. If ($\phi_u = 0$), the factor of safety is given by:

$$\text{F.S.} = \frac{c_u \cdot L_{arc}}{\sum \left\{W.\sin \alpha\right\}} \qquad \ldots\ldots\ldots\ldots\ldots (12.12)$$

A number of trial slip circles must be chosen so that the minimum factor of safety is determined. This method of analysis underestimates the factor of safety by 5 to 20 %.

12.5.2 Bishop's simplified solution

In this solution the vertical forces on the sides of each slices, Fig. 12.11, cancel each other, i.e.:

$$X1 - X2 = 0$$

The shear force on the base of slice is:

$$T = \frac{c'.\Delta L_{arc} + N'.\tan\phi'}{F.S.}$$

Resolving forces in the vertical direction:

$$W = N'.\cos\alpha + u.\Delta L_{arc}.\cos\alpha + \frac{c'.\Delta L_{arc}}{F.S.}.\sin\alpha + \frac{N'}{F.S.}.\tan\phi'.\sin\alpha$$

$$\therefore N' = \frac{W - \frac{c'.\Delta L_{arc}}{F.S.}.\sin\alpha - u.\Delta L_{arc}.\cos\alpha}{\cos\alpha + \frac{\tan\phi'.\sin\alpha}{F.S.}}$$

Putting $\Delta L_{arc} = b\sec\alpha$, and substituting in Equ. 12.10, after re-arrangement:

$$F.S. = \frac{1}{\sum W\sin\alpha}\sum\left[\frac{\{c'b + (W - ub)\tan\phi'\}\sec\alpha}{1 + \frac{\tan\alpha\tan\phi'}{F.S.}}\right] \quad \dots\dots\dots\dots\dots (12.13)$$

The pore pressure u at any point on the slip surface can be related to the total pressure at that point by means of the pore pressure raio r_u which is defined as:

$$r_u = \frac{u}{\gamma h} \quad \dots\dots\dots\dots\dots (12.14)$$

Hence, the factor of safety can be expressed as:

$$F.S. = \frac{1}{\sum W\sin\alpha}\sum\left[\frac{\{c'b + W(1 - r_u)\tan\phi'\}\sec\alpha}{1 + \frac{\tan\alpha\tan\phi'}{F.S.}}\right] \quad \dots\dots\dots\dots\dots (12.15)$$

Equs. (12.13) and (12.15) must be solved for (F.S.) by an iteration process. Using the computer, wider application for slopes having various geometrical shapes and soil conditions is provided.

12.6 The friction circle method

This method is suitable for solving simple cases in homogenous (c–ϕ) soils. The failure surface is assumed to be passing through the toe. In Fig. 12.12 a trial circle (AD) with radius (r) and center (O) is drawn. Considering the equilibrium of a slice weighing (ΔW), the soil resistance to sliding is offered by cohesive force $\{c_m \cdot \Delta L_{arc}\}$, where ($c_m$) is the mobilized cohesion along the slip surface, and frictional force (ΔR) inclined with angle (ϕ) to the normal to the circle at pint (E).

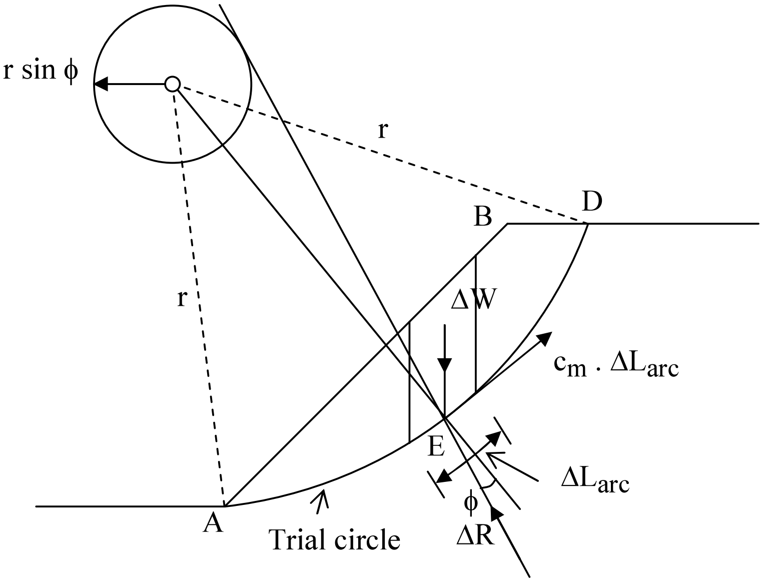


Fig. 12.12 The friction circle method

It is noticed that if a circle of radius ($r \cdot \sin\phi$) is draw concentric with the slip circle, all (ΔR) forces will be tangent to such circle, called the **friction circle**. The resultant (R) is approximately tangent to the friction circle.

Now, the total cohesive resistance ($c_m \cdot L_{arc}$) is the resultant of resistance along successive length elements (ΔL_{arc}). In Fig. 12.13, line (AD) of length (L) represents the magnitude and direction of the elementary cohesive forces. The line of action of the resultant of cohesive forces is at a distance (x) from the centre (O), where:

$$c_m \cdot L \cdot x = c_m \cdot L_{arc} \cdot r$$

$$\therefore\ x = r \cdot \frac{L_{arc}}{L} \qquad \text{… … … … … (12.16)}$$

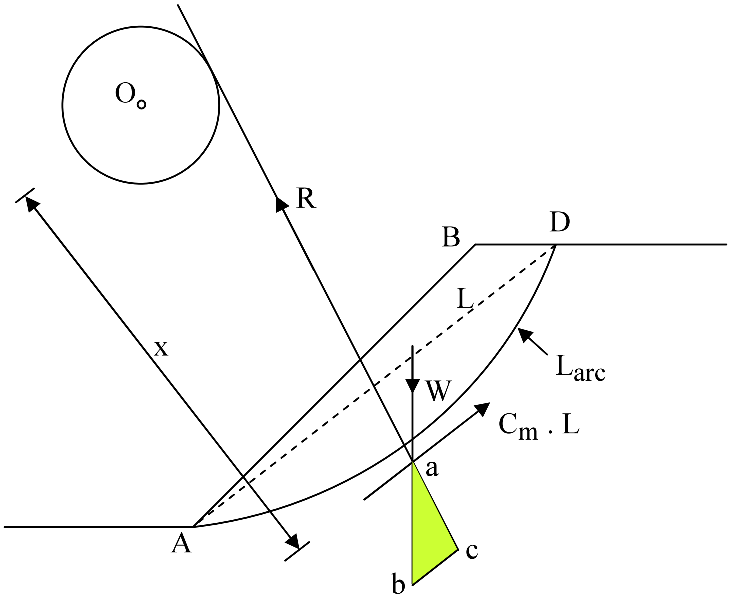


Fig. 12.13 Construction of the triangle of forces

From the equilibrium of the forces acting on the sliding mass, (R) is drawn tangent to the friction circle from the point of intersection of (W) and (c_m. L). A triangle of forces (abc) is drawn, where (ab) represents the total weight (W) from which (bc), representing (c_m. L) is obtained. The factor of safety with respect to cohesion (F_c), assuming fully mobilized frictional resistance is given by:

$$F_c = \frac{c}{c_m} \qquad \text{... ... ... (12.17)}$$

The mobilized cohesion can be calculated according to a pre-determined factor of safety with respect to friction factor of safety (F_ϕ) defined by:

$$F_\phi = \frac{\tan \phi}{\tan \phi_m} \qquad \text{... ... ... (12.18)}$$

where:

ϕ_m = mobilized angle of internal friction.

Several trials are usually made for each trial circle by varying (F_{ϕ}) value and determining the corresponding (F_c) value. The obtained values are plotted, as shown in Fig. 12.14, and equalization of both values is assumed to give the true factor of safety against sliding.

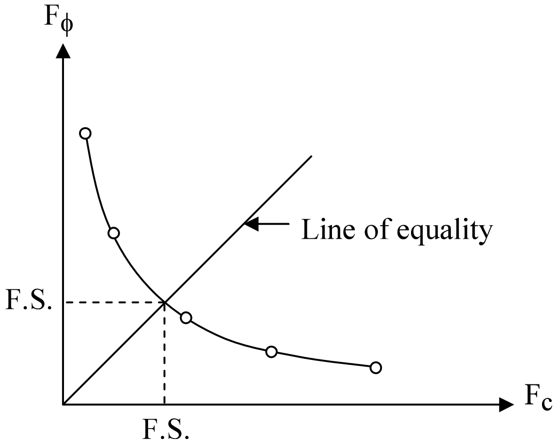


Fig. 12.14 Determination of the factor of safety

As stated before, several trial slip circles will finally enable determination of the most critical circle which yields the minimum factor of safety for the stability of the slope.

12.7 Taylor's stability number

Taylor developed a method for calculating the factor of safety with respect to cohesion by determining a dimensionless value called the "stability number" (S_n), where:

$$S_n = \frac{c_m}{\gamma H} \qquad \text{… … … … … (12.19)}$$

The factor of safety with respect to cohesion is given by:

$$F_c = \frac{c}{c_m} \qquad \text{… … … … … (12.٢٠)}$$

The relations between (S_n), slope angle (β) and (ϕ) are given in Fig. 12.15. Also, in the same plot, for soils having $\phi = 0$, the (S_n) values are given for different values of depth factor (D), where (D) is defined as shown.

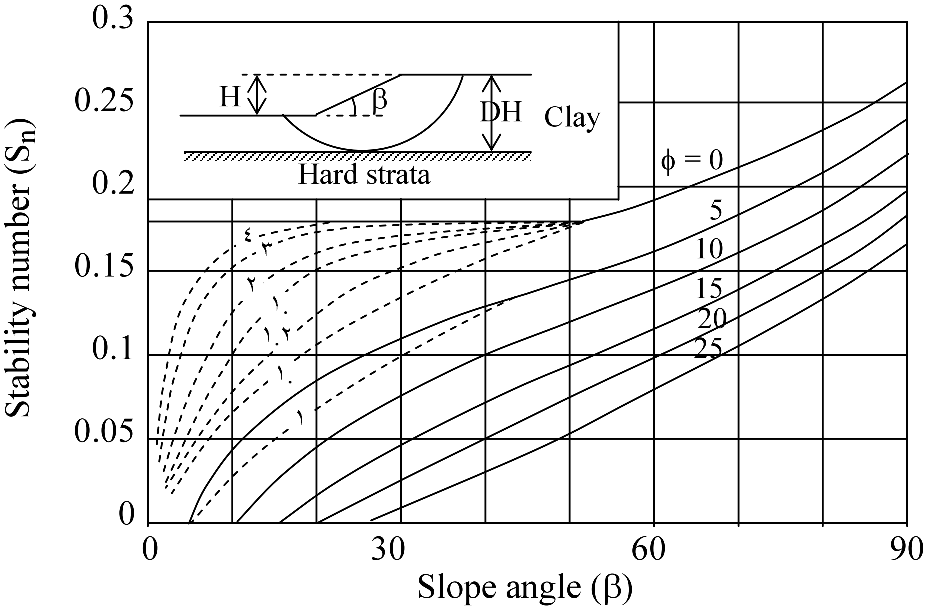


Fig. 12.15 Taylor's stability curves

Several trials are made by varying (Fϕ) value, calculating (ϕ_m) from Equ. (12.18), and using it as (ϕ) in Fig. 12.15. The value of (S_n) is determined and the corresponding (F_c) value is calculated. The obtained values of (Fϕ) and (F_c) are ploted, as shown in Fig. 12.14. Equalization of both values is assumed to given the true factor of safety against sliding.

12.8 Position of the critical circle

In cases where the critical circle is likely to pass through the toe of the slope, the position of its centre can be determined as given in Fig. 12.16 and Table 12.2.

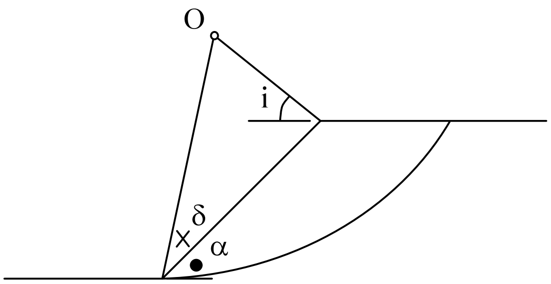


Fig. 12.16 Position of the critical toe circle for ϕ = 0 soil

Table 12.2 Values of i and δ angles

Slope	α	δ	i
1 : 1	45	28	٣٧
1 : 1.5	33.8	٢٦	٣٥
1 : 2	26.6	٢٥	٣٥
1 : 3	18.4	٢٥	٣٥
1 : 5	11.3	٢٥	٢٧

When slip occurs along a circle below the toe, which is termed **base failure**, the center of this circle is located on the vertical through the mid point of the slope. The central angle of the sector is 133.5^{o}, Fig. 12.17. This case usually occurs for flat slopes, or when the shear strength of the foundation soil is relatively low.

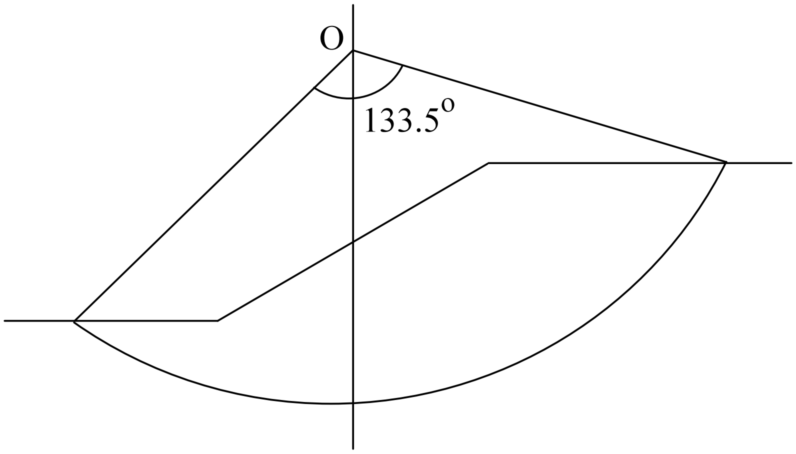


Fig. 12.17 Location of the critical circle for base failure

12.9 Wedge method of slope stability analysis

In many slope stability problems it is noticed that the actual failure surface is close to a plane, or a number of planes. This situation arises when there are weak start within or beneath the slope, also when the slope rests upon a very strong stratum. Fig. 12.18 shows a case for which the wedge method is applicable.

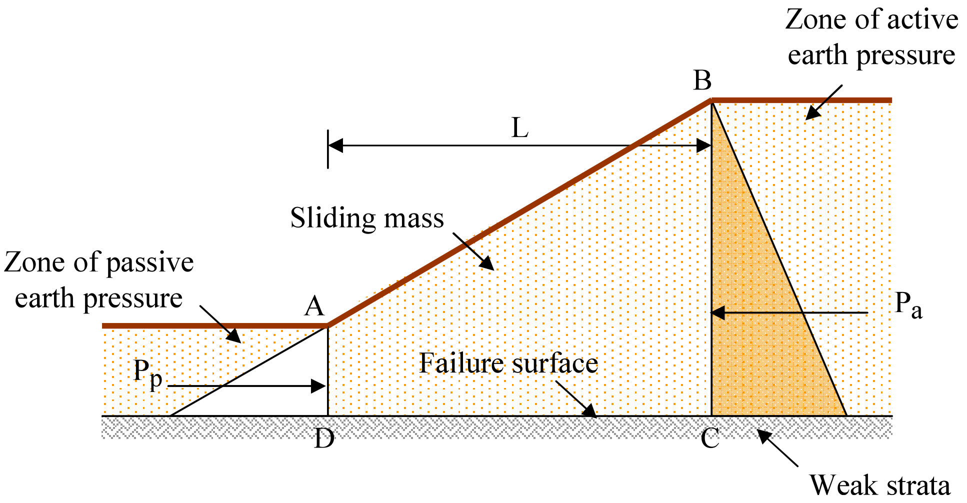


Fig. 12.18 Translational slip on weak stratum

Stability of the slope is checked with respect to sliding of the soil mass ABCD. The failure surface CD is developed along the top of the weak strata. The soil at the higher level will exert an active earth pressure (P_a) on the wedge ABCD, and at the lower level a passive earth pressure (P_p). The net driving forces is:

Driving force = $P_a - P_p$

Assuming that the weak strata is purely cohesive:

Resisting force = c_u. L

$$\therefore \text{ F.S.} = \frac{c_u \, L}{P_a - P_p} \qquad \dots\dots\dots\dots\dots (12.21)$$

where: c_u = undrained cohesion of the weak strata

P_a = active earth pressure

P_p = passive earth pressure

(N.B.: Methods for calculating earth pressure are given in the next chapter)

12.10 Protection and treatment of slopes

If a slope is exposed to external effects causing instability, it is common to protect it by adopting one or more of the following procedures: (Fig. 12.19)

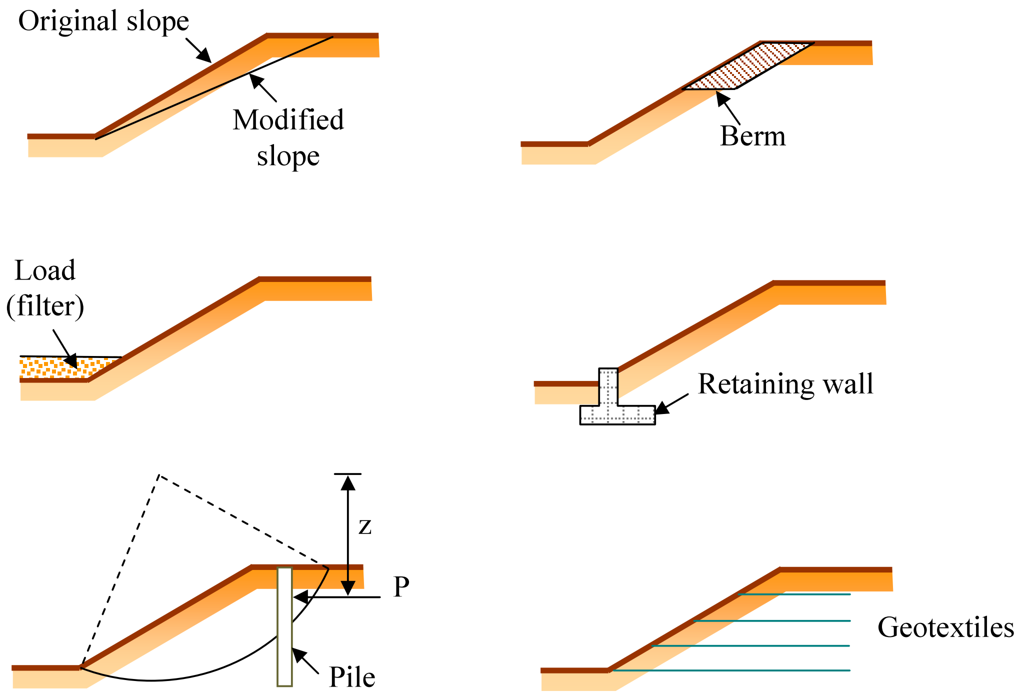


Fig. 12.19 Treatment of slopes

(a) Covering the slope by a layer of broken stones or coarse gravel, usually 30 cm thick. This helps for protection against the action of wind and rain.

(b) Using vegetation so that plant roots binds loose soil materials at the top of the slope.

(c) Covering the slope by large block of rock over a suitable filter.

(d) Covering the slope by pitching in mortar over a suitable filter.

(e) Using reinforced concrete lining over a suitable filter.

If the factor of safety for stability of a slope unsatisfactory, it is common to treat it by following one or more of the following procedures:

(a) Change of profile by lowering of a slope, or by adding berm(s).

(b) Using additional load at the toe of the slope.

(c) Using retaining structure at the toe of the slope.

(d) Installing piles to interrupt the critical failure surface. (Added resisting moment = P . z)

(e) Using textiles which reinforce the soil mass.

12.11 Examples

(1) A cutting is made in a granular soil whose angle of friction is 35^{o}. Find the safe slope angle for a factor of safety of 1.5. Check the safety of the slope when exposed to seepage parallel to it. Take $\gamma_{sat.} = 2.03\ t/m^3$.

Solution:

$$F.S. = \frac{\tan\phi}{\tan\beta}$$

$$1.5 = \frac{\tan 35}{\tan\beta} \qquad \therefore \beta = 25^{o}$$

Considering seepage :

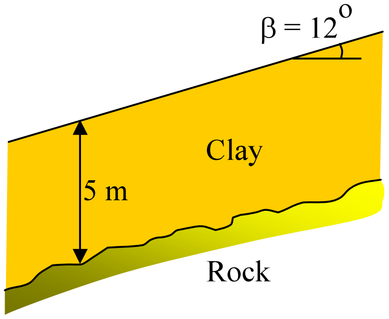
$$F.S. = \frac{\tan\phi}{\tan\beta}\cdot\frac{\gamma_{sub.}}{\gamma_{sat.}} = \frac{\tan 35}{\tan 25}\cdot\frac{2.03-1}{2.03} = 0.7\ (\text{unsafe})$$

(2) For the shown soil formation, find the factor of safety for stability of the proposed slope. Data:

$$\gamma = 1.8\ t/m^3$$

$$c' = 2.75 - 1.3\ z + 0.3\ z^2$$

$$\phi' = 8^{o}$$



Solution:

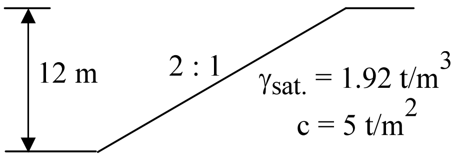
$$F.S. = \frac{c' + \gamma\, z \cos^2\beta \tan\phi'}{\gamma\, z \sin\beta \cos\beta}$$

$$= \frac{2.75 - 1.3\,z + 0.3\,z^2 + 1.8\,z\cos^2 12 \tan 8}{1.8\,z \cos 12 \sin 12}$$

z (m)	c' (t/m^2)	F.S.
1	1.75	3.74
2	1.35	1.71
3	1.55	1.41
4	2.35	1.54
5	3.75	1.85

بتغيير قيمة z من ١ الى ٥
والتعويض فى افلعلاقة اعلاه
نحصل على قيم .F.S المختلفة
ومنها نوجد القيمة الادنى وهى
1.41
ويلاحظ اننا اذا غيرنا قيم z كل
0.01 m يمكن ان نحصل على
قيمة ادنى ادق لـ .F.S

(3) For the shown cutting and soil formation, find the factor of safety against circular slip. Consider the $\phi = 0$ condition. Assume that the slip circle passes through the toe of the slope. Consider the effect of the tension crack, unfilled with water.

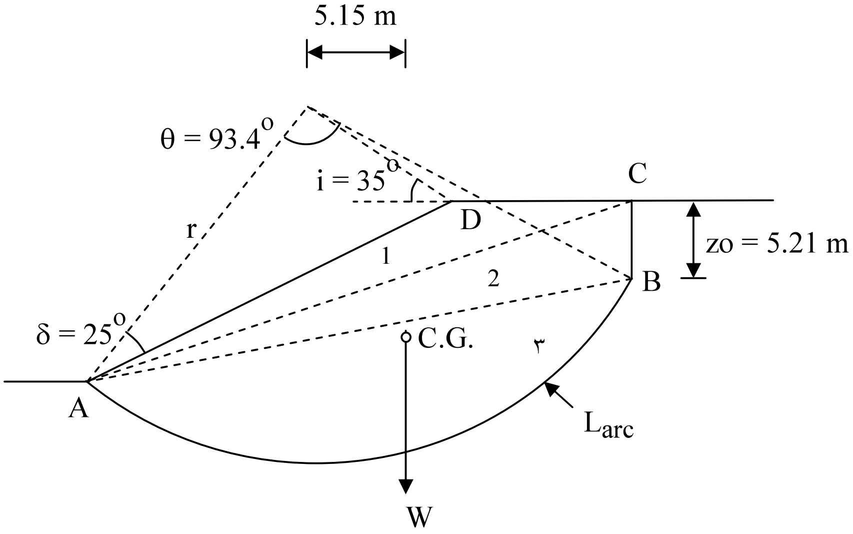


Solution:

From Table 12.2: $\delta = 25^o$, $i = 35^o$

$z_o = 2\,c / \gamma = 2 \times 5 / 1.92 = 5.21$ m

The centre of the critical circle is determined as shown and the slip arc AB is drawn.



Areas 1, 2, 3 are calculated and the C.G. of ABCD is located.

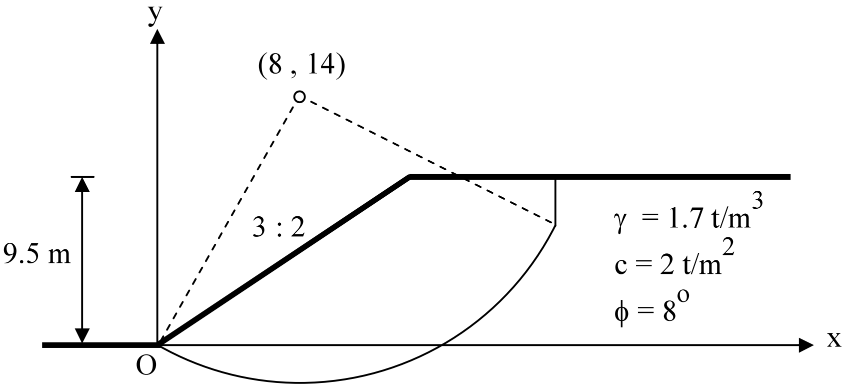
Area ABCD = 364 m^2,   W = 346 x 1.92 = 6.99.15 t/m'

$\theta = 93.4^o$ r = 23.64 m

$$L_{arc} = 93.4 \times \frac{\pi}{180} \times 23.61 = 38.55 \text{ m}$$

$$\text{F.S.} = \frac{5 \times 38.55 \times 23.64}{699.15 \times 5.2} = 1.27$$

(4) For the shown cutting in homogeneous silty clay, determine the factor of safety against slip along the given circular arc. Allow for development of tension cracks. Use the slices method, Fellenius solution.



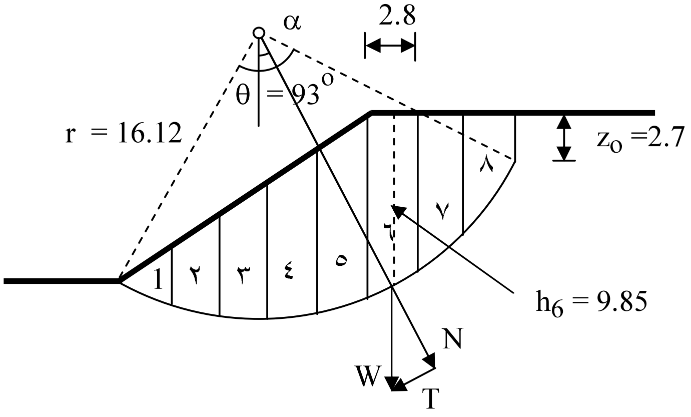
Solution:

$$zo = \frac{2c}{\gamma\sqrt{k_a}}$$

$$ka = \frac{1-\sin\phi}{1+\sin\phi} = 0.76$$

$$zo = \frac{2\times 2}{1.7\sqrt{0.76}} = 2.7\text{ m}$$

Divide the sliding wedge into 8 slices: $b = 2.8$ m



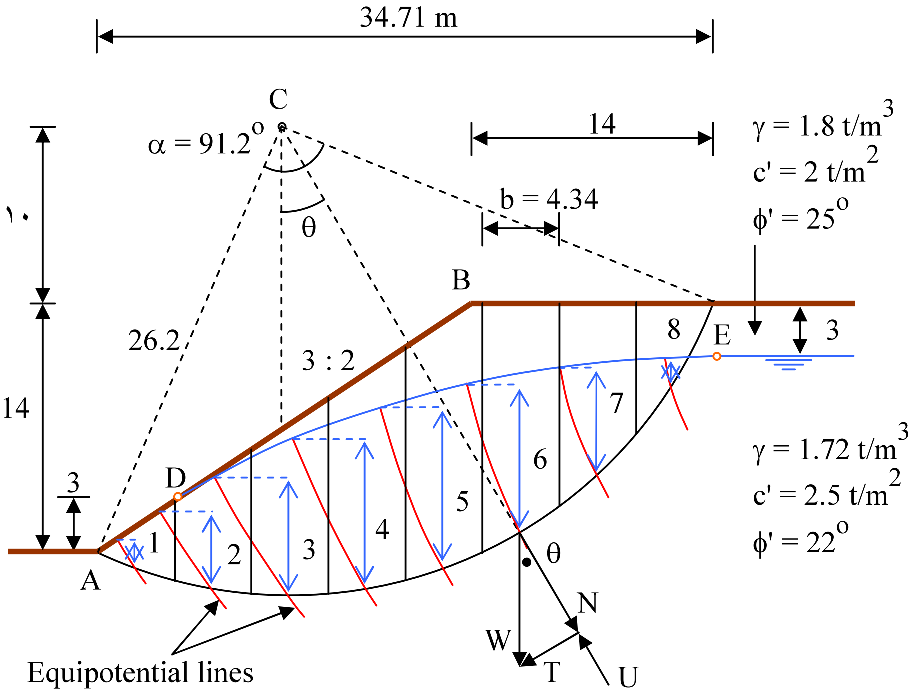
Slice No.	W t/m'	α Deg.	N t/m'	T t/m'
1	7.7	-22.5	7.1	-2.95
2	21.3	-13.5	20.7	-4.97
3	32.2	-3.5	1.1	-1.97
4	40.8	6.5	40.5	4.6
5	47.0	16.5	45.1	13.3
6	46.8	27.5	41.5	21.6
7	38.0	39.5	29.3	24.2
8	23.9	54	14	19.3
			Σ = 229.3	Σ = 73.1

$$L_{arc} = \frac{93}{180}\times\pi\times 16.12 = 26.2\text{ m}$$

$$\therefore \text{F.S.} = \frac{2\times 26.2 + 229.3\tan 8}{73.1} = 1.16$$

(5) The shown cutting is exposed to groundwater seepage represented by the seepage line DE. Soil data and flow net are given. Use the slices method (Fellenius solution) to calculate the factor of safety against slip along the given circular arc. Take F at mid point of the slope AB.

Solution:



Divide the sliding wedge into 8 slices (b = 4.34 m). For each slice calculate $W = \Sigma \gamma \cdot h$, measure h_w and calculate: $U = (h_w \cdot \gamma_w) \times b/\cos\theta$.

Slice No.	W t/m'	T t/m'	N t/m'	U t/m'	N' t/m'
1	22.4	-6.5	21.4	13.6	7.8
2	46.5	-7.1	46.0	26.2	19.8
3	70.7	0.9	70.7	33.4	37.3
4	89.6	16.0	88.2	36.7	51.5
5	103.0	35.5	96.7	38.0	58.7
6	95.0	48.4	81.7	37.1	44.6
7	70.6	47.8	52.1	31.1	21.0
8	40.1	32.6	23.3	14.2	9.0
	Σ =	167.6			249.7

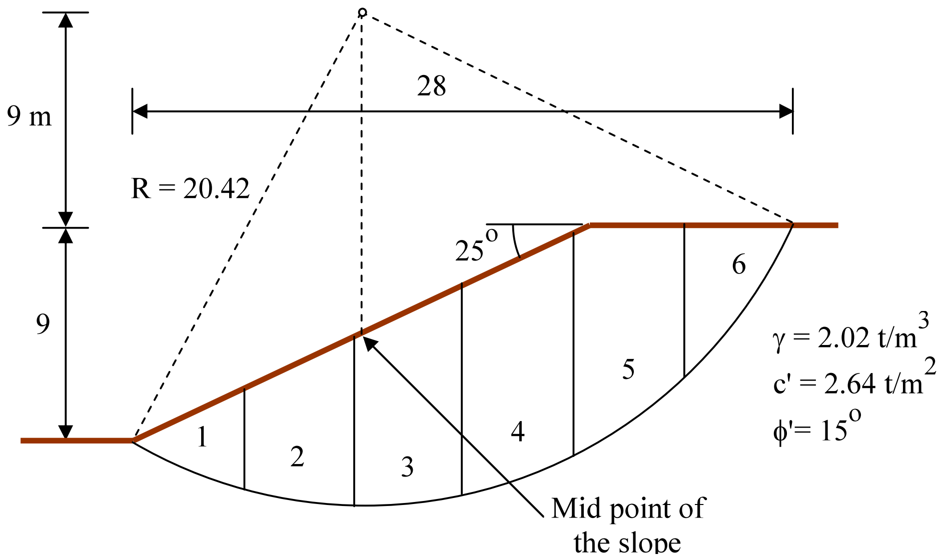
$\alpha = 91.2^{o}$

$L_{arc} = 41.7$ m.

Consider that the bottom ends of slices 1 to 8 are located in the lower soil layer.

$$F.S. = \frac{2.5 \times 41.7 + 249.7 \times \tan 22}{167.6} = 1.22$$

(6) The shown embankment has just been constructed. Find the factor of safety against slip along the shown trial circular arc. Take average pore water pressure ratio = 0.5. Use Bishop's method.



Solution:

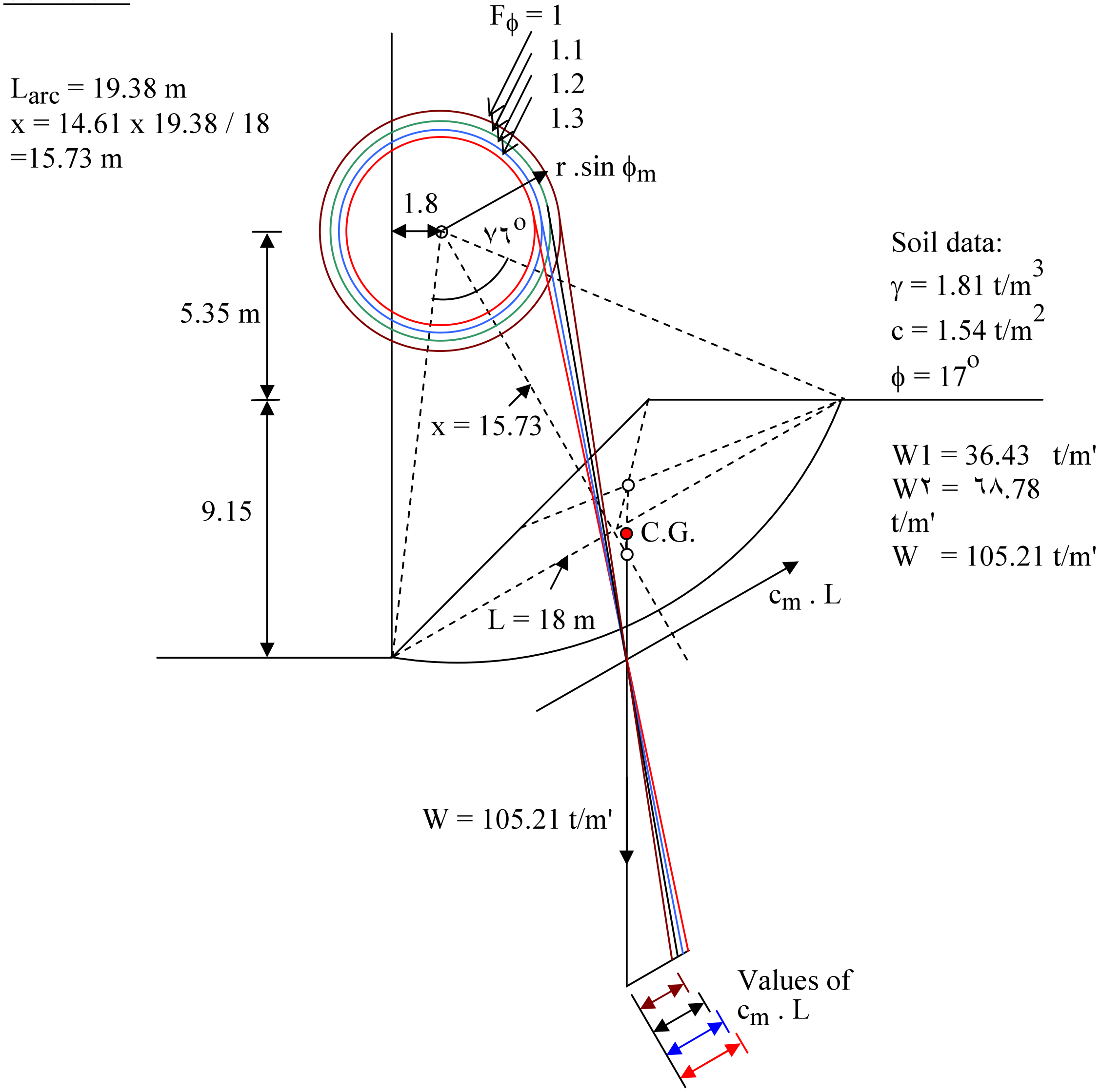
Divide the sliding wedge into 6 slices (b = 4.66 m). Complete the table as shown:

Slice No.	W t/m'	α deg.	W . sin α t/m'	$\frac{\{c'b + W(1 - ru)\tan\phi'\}\sec\alpha}{1 + \frac{\tan\alpha\tan\phi'}{F.S.}}$		
				F.S. = 1	F.S. = 1.416	F.S. = 1.444
1	19.76	-18.68	-6.34	17.36	16.87	16.84
2	51.93	-7.47	-6.75	20.14	19.93	19.92
3	73.12	5.64	7.19	21.64	21.6	21.81
4	83.98	19.07	27.44	22.81	23.4	23.43
5	75.25	33.71	41.77	22.83	23.9	23.95
6	32.6	48.18	24.3	19.25	20.65	20.72
			$\Sigma =$ 87. 63	124.06	126.571	126.7

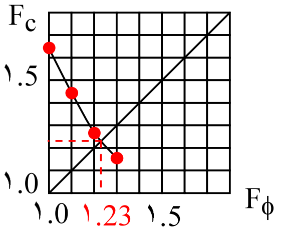
F.S. 1st trial = 1
F.S. 2nd trial = 124.06/87.63 = 1.416
F.S. 3rd trial = 126.571/87.63 = 1.444
F.S. 4th trial = 126.7/87.63 = 1.445
Take F.S. = 1.44

(7) For the shown cutting and soil formation, use the friction circle method to find the factor of safety against slip along the given trial circular arc.

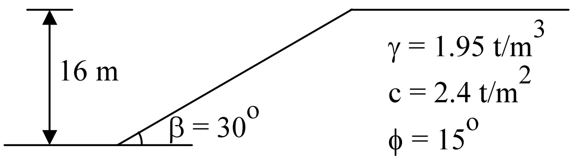
Solution:



F_ϕ	ϕ_m	r.sin ϕ_m	c_m . L	F_c
١	17	4.27	١٧	١.64
1.1	15.53	3.91	19.33	1.44
1.2	14.29	3.61	21.67	1.28
1.3	13.23	3.34	24	1.16



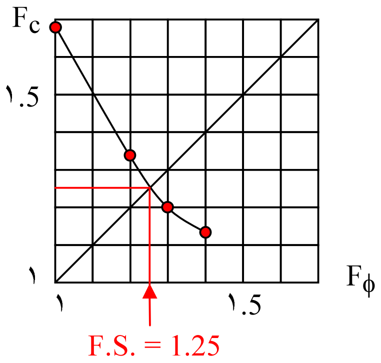
(8) For the shown cutting and soil data, find the factor of safety against slip. Assume $F_c = F_\phi$. Use Taylor's stability number method.



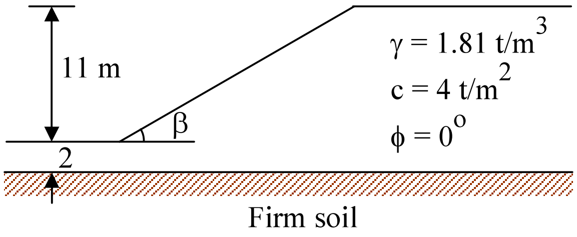
Solution:

Use the curves given in Fig. 12.15.

F_ϕ	ϕ_m	S_n	c_m	F_c
١	15	0.046	1.43	١.68
1.2	12.6	0.058	1.81	1.33
1.3	11.6	0.064	2	1.2
1.4	10.8	0.068	2.12	1.13



(9) For the shown cutting and soil data, find the slope angle (b) such that the factor of safety against slip is 1.5.



Solution:

Use the curves given in Fig. 12.15.

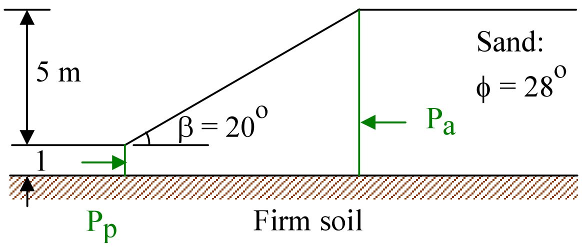
$$H.D = 13\,m, \quad H = 11\,m \quad \therefore D = 1.18$$

$$c_m = \frac{4}{1.5} = 2.67\,t/m^2$$

$$S_n = \frac{c_m}{\gamma \cdot h} = \frac{2.67}{1.81 \times 11} = 0.134$$

$$\text{From curves}: \beta = 22^o$$

(10) For the shown cutting and soil data, find the factor of safety against slip. Take P_a = 11.7 t/m' and P_p = 2.5 t/m'



Solution:

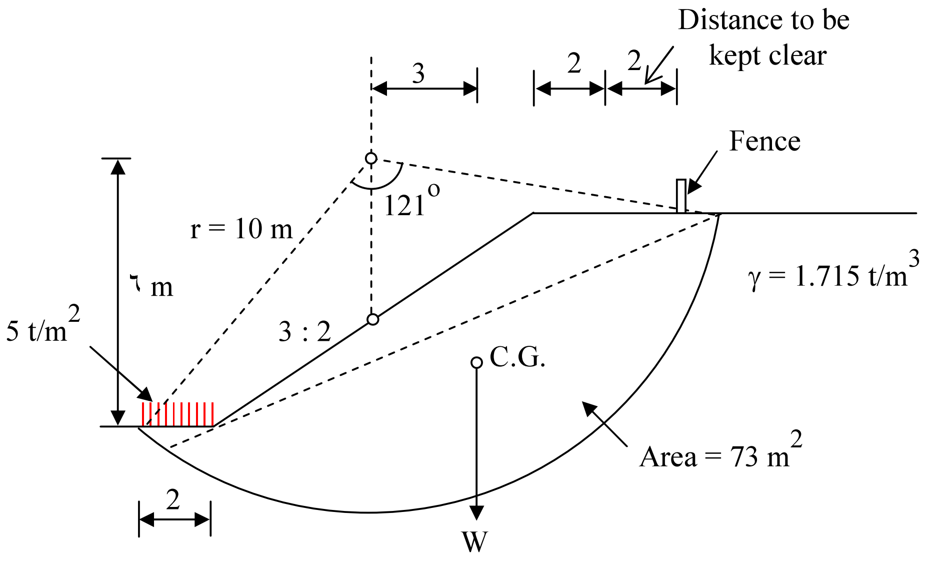
$$\text{For sand: F.S.} = \frac{\tan 28}{\tan 20} = 1.46$$

For sand-clay inteface:

$$\text{F.S.} = \frac{1.1 \times 5 \cot 20}{11.7 - 2.5} = 1.64$$

$$\therefore \text{F.S. against slip} = \text{F.S.}_{\text{min.}} = 1.46$$

(11) The shown cutting shows signs of failure along the indicated slip surface. Remedial measures are therefore required. The site is limited by a trench, and at least 2 m clearance is required inside this fence. Material can be stored at the base of the cutting giving stress of 5 t/m^2. Assuming $\phi = 0$ condition and no tension cracks have formed, suggest the suitable protections for the slope.



Solution:

$$W = 1.715 \times 73 = 125.195 \text{ t/m'}$$

$$L_{arc} = \frac{121}{180} \times \pi \times 10 = 21.12 \text{ m}$$

Take F.S. = 1 (Slope is just slipping)

$$\therefore 1 = \frac{c \times 21.12 \times 10}{125.195 \times 3} \quad \therefore c = 1.78 \text{ t/m}^2$$

$$\text{Re sisting moment} = 21.12 \times 10 \times 1.78 = 375.6 \text{ t.m/m'}$$

$$\text{Driving moment} = 125.195 \times 3 = 375.6 \text{ t.m/m'}$$

(a) Try 5 t/m^2 at the base of the slope :

$$\therefore \text{Added resisting moment} = 5 \times 2 \times 5.5 = 55 \text{ t.m/m'}$$

$$\therefore \text{F.S.} = \frac{375.6 + 55}{375.6} = 1.15$$

(b) To increase safety try cutting a berm as shown :

$$\text{Reduced driving moment} = 2 \times 3 \times 1.715 \times 3.25 = 33.44 \text{ t.m/m'}$$

$$\therefore \text{F.S.} = \frac{375.6 + 55}{375.6 - 33.4} = 1.26$$

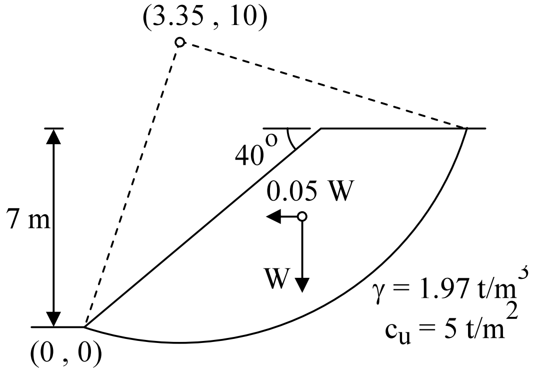
12.12 Problems

(1) A cutting is made in a granular soil whose angle of friction is 35^o. Find the safe slope angle for a factor of safety of 1.5. Check the safety of the slope when exposed to seepage parallel to it. Take $\gamma_{sat} = 2.03$ t/m^3.

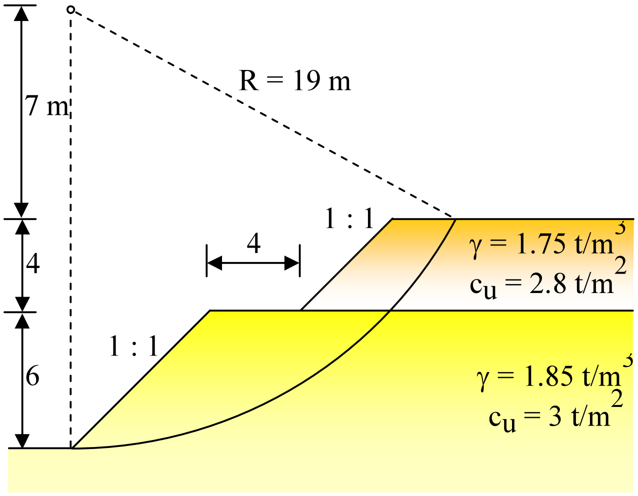
(2) For a clay formation with ground surface sloping at 10^o, find the factor safety against slope failure at depths 1, 2, 3, 4, 5 m.
Data:

$\gamma = 1.66$ t/m^3
$c' = 2$ t/m^2
$\phi' = 70$

(3) The figure shows a cutting with a slope of 40^o made in a clay soil. For the shown slip circle, find the factor of safety against slope failure. Find also the factor of safety in the case of earthquake with acceleration in the horizontal direction 0.05 times the gravitational acceleration.

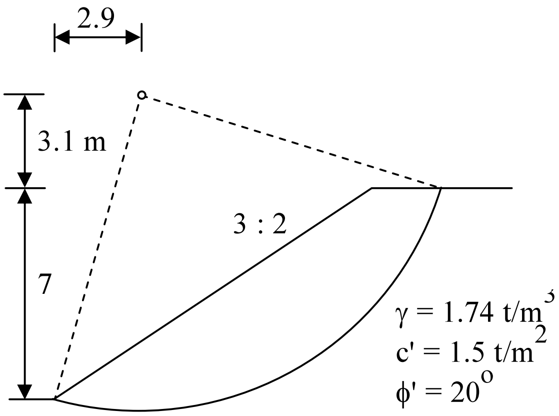


(4) For the shown cutting and soil formation, find the factor of safety against slip failure along the given trial slip circle. Take into consideration the effect of tension crack.

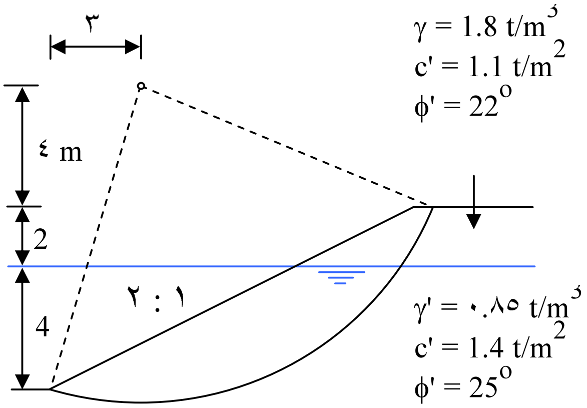


(5) a- For the shown cutting, use the method of slices to determine the factor of safety against slip failure. The slip circle giving the minimum factor of safety is shown.

b - Resolve Problem (5-a) using Taylor's method, and compare between the results.

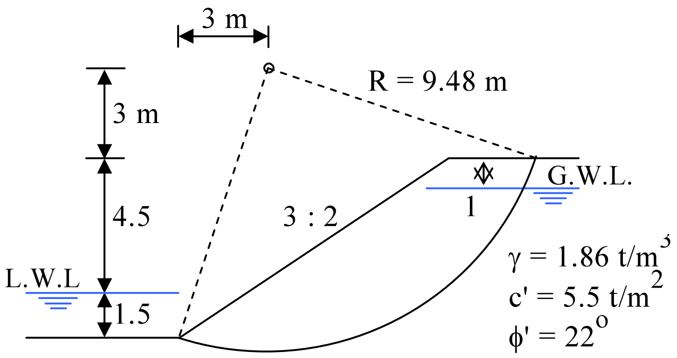


(6) For the shown soil formation and cutting, use the method of slices to determine the factor of safety against slip failure.

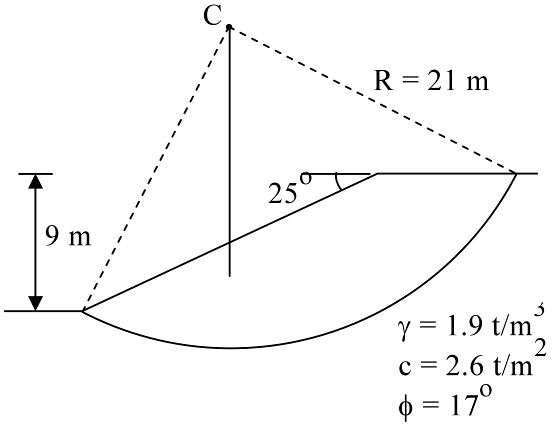


(7) The shear strength parameters of a soil layer are : $c = 1.7$ t/m^2 and $\phi = 10^o$. A sloping cut, 10 m high, is planned to be made in this soil layer. What inclination is required to achieve a factor of safety of 1.25.
Take $\gamma = 1.77$ t/m^3.

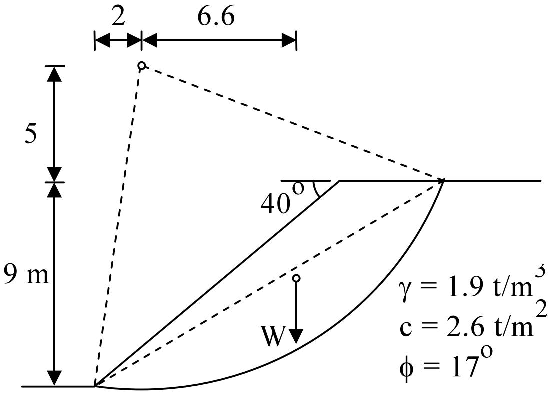
(8) For the shown side slope of a canal, find the factor of safety against slip along the shown circular arc. Consider the case of rapid draw down of water level in the canal.



(9) An embankment 9 m high is required to be constructed at a slope of 25° to the horizontal. Use Bishop's simplified solution to find the factor of safety against slip failure along the shown trial slip circle for the case of just after construction. Take pore-pressure ratio $r_u = 0.6$.

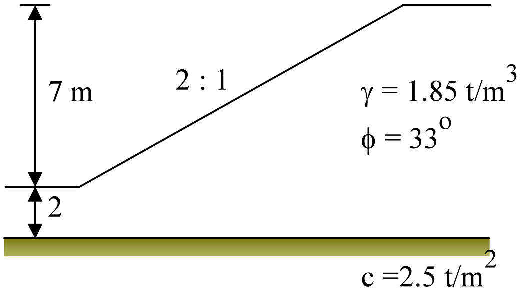


(10) The section of an embankment is shown in figure. For the given assumed slip surface use the friction circle method to determine the factor of safety with respect to cohesion, and also the factor of safety assuming this to be the same for both cohesion and frictional resistance. Neglect the effect of tension cracks.



(11) A cutting of depth 10 m is to be made in a clay soil of which the density is t/m^3 and cohesion 4 t/m^2. There is a hard layer under the clay at 13 m below the original ground surface. Assuming $\phi = 0$, and allowing for a factor of safety of 1.5, find the slope of the cutting. Use Taylor's stability curves.

(12) The shown cutting is to be made in a soil whose properties are as given. There is a weak soil layer at the given depth. Check the stability of the cutting with respect to sliding along the top of the weak soil layer.



* * *